1

MEASUREMENT AND GEOMETRY

PYTHAGORAS' THEOREM

Pythagoras' theorem expresses the relationship between the sides of a right-angled triangle. It is used in the building and construction industries to ensure right angles in building frames and to calculate the lengths of building materials. It is also used in Geography and GPS systems when calculating distances, and in checking for corrupted data sent electronically. Pythagoras had no idea how useful his theorem would be!

Chapter outline

	Working mathematically				
1.01 Square roots and surds	U	F			
1.02 Discovering Pythagoras' theorem	U	F		R	C
1.03 Finding the hypotenuse	U	F			
1.04 Finding a shorter side	U	F			
1.05 Hypotenuse or shorter side?	U	F			
1.06 Testing for right-angled triangles	U	F	PS	R	C
1.07 Pythagorean triads	U	F			
1.08 Pythagoras' theorem problems	U	F	PS		

This is a Year 9 topic in the Australian Curriculum but a Stage 4 (Years 7–8) topic in the NSW syllabus.

Wordbank

converse A rule or statement turned back-to-front; the reverse statement

hypotenuse The longest side of a right-angled triangle; the side opposite the right angle

Pythagoras An ancient Greek mathematician who discovered an important formula about the sides of a right-angled triangle

surd A square root (or other type of root) whose exact decimal value cannot be found

theorem A formal rule or a formula

triad A group of 3 related objects

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

In this chapter you will:

- investigate square roots, surds and irrational numbers
- investigate the relationship between the sides of a right-angled triangle
- solve problems involving Pythagoras' theorem, writing the answer as a decimal or as a surd
- test whether a triangle is right-angled by using the converse of Pythagoras' theorem
- investigate Pythagorean triads

SkillCheck ANSWERS ON P. 544

1 Evaluate each expression.

a 4^2 **b** 12^2 **c** 30^2 **d** 2.1^2

e 10.3^2 **f** $3^2 + 4^2$ **g** $5^2 + 12^2$ **h** $5.4^2 - 3.2^2$

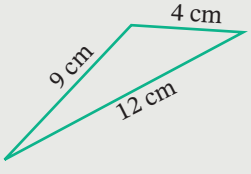
2 Evaluate each square root.

a $\sqrt{16}$ **b** $\sqrt{64}$ **c** $\sqrt{100}$ **d** $\sqrt{196}$

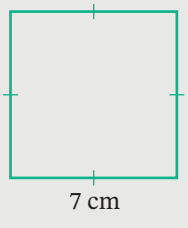
e $\sqrt{121}$ **f** $\sqrt{49}$ **g** $\sqrt{1.21}$ **h** $\sqrt{2.25}$

3 Find the perimeter of each shape.

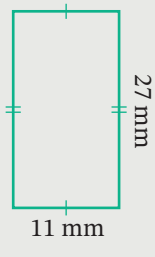
a



b

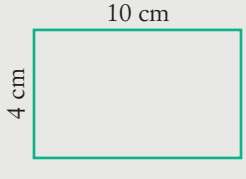


c

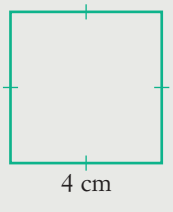


4 Find the area of each shape.

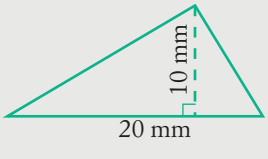
a



b



c



5 Write each number correct to one decimal place.

a 10.33 **b** 4.67 **c** 7.654 **d** 0.8888

6 Evaluate each expression correct to one decimal place.

a $\sqrt{3^2 + 7^2}$ **b** $\sqrt{5.6^2 + 9.2^2}$ **c** $\sqrt{15^2 - 8^2}$

Square roots and surds

1.01

The **square root** ($\sqrt{\ }$) of a number is the **positive** value which, if squared, will give that number. For example:

- $\sqrt{25} = 5$ because $5^2 = 25$ 'the square root of 25'
- $\sqrt{49} = 7$ because $7^2 = 49$ 'the square root of 49'

Most square roots do not give exact answers like the ones above. For example, $\sqrt{10} = 3.16227766\ldots \approx 3.2$. Such square roots are called **surds**. A surd is a square root ($\sqrt{\ }$), cube root ($\sqrt[3]{\ }$), or any other type of root whose exact decimal or fraction value cannot be found. As a decimal, its digits run endlessly *without repeating*, so they are *neither* terminating nor recurring decimals. A surd cannot be written in fraction form $\frac{a}{b}$ so it is also called an **irrational** number.

WS Table of squares and square roots
1.01
WS Surds

Example 1

Evaluate each expression correct to 2 decimal places.

a $\sqrt{7^2+9^2}$ **b** $\sqrt{12^2-10^2}$

Solution

a $\sqrt{7^2+9^2} = 11.40175\ldots$
≈ 11.40

On a calculator, enter $\sqrt{\ }$ (7 x^2 + 9 x^2) =

OR:

$7^2 + 9^2 = 130$

On a calculator, enter 7 x^2 + 9 x^2 =

$\sqrt{130} = 11.40175\ldots$
≈ 11.40

On a calculator, enter $\sqrt{\ }$ Ans =

b $\sqrt{12^2-10^2} = 6.63324\ldots$
≈ 6.63

On a calculator, enter $\sqrt{\ }$ (12 x^2 − 10 x^2) =

OR:

$12^2 - 10^2 = 44$

On a calculator, enter 12 x^2 − 10 x^2 =

$\sqrt{44} = 6.63324\ldots$
≈ 6.63

On a calculator, enter $\sqrt{\ }$ Ans =

Example 2

Select the surds from this list of square roots:

$\sqrt{72}$ $\sqrt{121}$ $\sqrt{64}$ $\sqrt{90}$ $\sqrt{28}$

Solution

$\sqrt{72} = 8.4852...$ $\sqrt{121} = 11$ $\sqrt{64} = 8$

$\sqrt{90} = 9.4868...$ $\sqrt{28} = 5.2915...$

so the surds are $\sqrt{72}$, $\sqrt{90}$ and $\sqrt{28}$.

EXERCISE 1.01 ANSWERS ON P. 544

Square roots and surds U F

EXAMPLE 1

1 Evaluate each square root.

a $\sqrt{196}$ **b** $\sqrt{900}$ **c** $\sqrt{64}$

d $\sqrt{625}$ **e** $\sqrt{121}$ **f** $\sqrt{361}$

2 Evaluate each square root, correct to 2 decimal places.

a $\sqrt{12}$ **b** $\sqrt{45}$ **c** $\sqrt{1001}$ **d** $\sqrt{325}$

e $\sqrt{153}$ **f** $\sqrt{207}$ **g** $\sqrt{98}$ **h** $\sqrt{888}$

i $\sqrt{24}$ **j** $\sqrt{110}$ **k** $\sqrt{297}$ **l** $\sqrt{689}$

3 Evaluate each expression, correct to 2 decimal places.

a $\sqrt{12^2 - 5^2}$ **b** $\sqrt{5^2 + 7^2}$ **c** $\sqrt{6^2 + 11^2}$

d $\sqrt{17^2 - 12^2}$ **e** $\sqrt{8^2 - 3^2}$ **f** $\sqrt{4^2 + 9^2}$

g $\sqrt{(1.5)^2 + (4.2)^2}$ **h** $\sqrt{(12.5)^2 - (7.1)^2}$ **i** $\sqrt{(25.7)^2 + (18.2)^2}$

EXAMPLE 2

4 Which of the following is **not** a surd? Select the correct answer **A**, **B**, **C** or **D**.

A $\sqrt{10.24}$ **B** $\sqrt{24}$ **C** $\sqrt{1000}$ **D** $\sqrt{3}$

5 Select all the surds from this list of square roots.

$\sqrt{98}$ $\sqrt{9}$ $\sqrt{225}$ $\sqrt{160}$ $\sqrt{36}$ $\sqrt{52}$

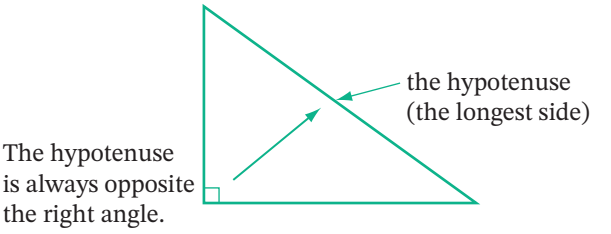
$\sqrt{144}$ $\sqrt{77}$ $\sqrt{18}$ $\sqrt{196}$ $\sqrt{200}$ $\sqrt{81}$

1.02 Discovering Pythagoras' theorem

WS A page of right-angled triangles

WS Proving Pythagoras theorem

A **right-angled triangle** has one right angle (90°) and 2 smaller angles. The side opposite the right angle is the *longest* side and is called the **hypotenuse**.



Pythagoras' theorem is a rule that describes the relationship between the hypotenuse and the other 2 (shorter) sides.

Example 3

a Construct a right-angled triangle with the 2 shorter sides being of length 3 cm and 4 cm.

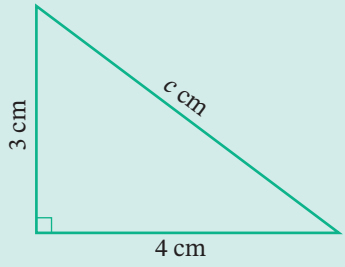
b Measure the length of the hypotenuse.

c Evaluate $3^2 + 4^2$.

d If c is the length of the hypotenuse in centimetres, evaluate c^2.

Solution

a



b Measuring with a ruler, the hypotenuse is 5 cm long.

c $3^2 + 4^2 = 25$

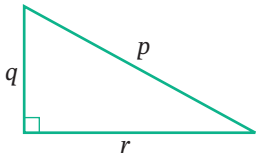
d $c^2 = 5^2 = 25$

EXERCISE 1.02 ANSWERS ON P. 544

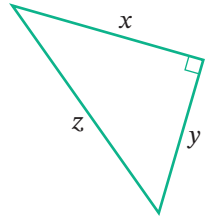
Discovering Pythagoras' theorem UFRC

1 Name the hypotenuse in each triangle. C

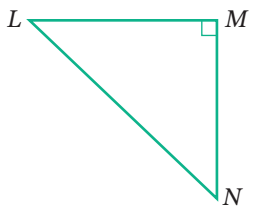
a



b



c



2 **a** Draw a right-angled triangle with short sides measuring 5 cm and 12 cm.

b Measure the length of the hypotenuse.

c Evaluate $5^2 + 12^2$.

d If c is the length of the hypotenuse in centimetres, evaluate c^2.

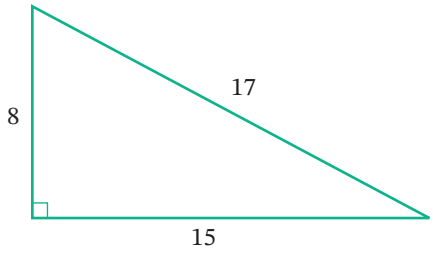
e Copy and complete each statement. C

The square of the hypotenuse = ____^{2} = ____

The sum of the squares of the 2 shorter sides = ____^{2} + ____^{2} = ____

EXAMPLE 3

3 Which equation is true for the right-angled triangle shown? Select the correct answer **A**, **B**, **C** or **D**. C



A $8 + 15 = 17$

B $8^2 + 15^2 = 17^2$

C $8^2 + 17^2 = 15^2$

D $17^2 + 15^2 = 8^2$

Foundation | Standard | Complex

4 **a** Use dynamic geometry software (or pencil, ruler and 5 mm grid paper) to draw a right-angled triangle with the shorter sides of length 6 cm and 8 cm, then measure the length of the hypotenuse.

b Copy and complete this table by:

- constructing each right-angled triangle with the measurements given for the 2 shorter sides a and b (as you did in part **a**)
- measuring the length, c, of the hypotenuse
- evaluating the values of a^2, b^2, $a^2 + b^2$, c^2 C

Shorter sides (cm)		Hypotenuse (cm)				
a	b	c	a^2	b^2	$a^2 + b^2$	c^2
6	8					
2.5	6					
1.5	2					
4	7.5					

5 What rule or relationship did you discover between the length of the hypotenuse and the lengths of the other 2 sides? R C

1.03 Finding the hypotenuse

Pythagoras' theorem

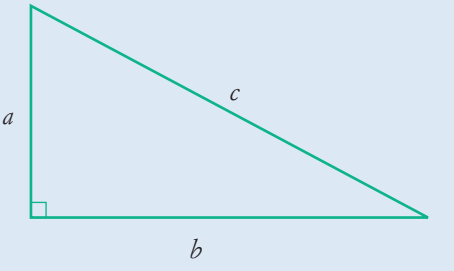
Pythagoras' theorem

Pythagoras' theorem

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other 2 sides.

If c is the length of the hypotenuse, and a and b are the lengths of the other 2 sides, then:

$$c^2 = a^2 + b^2$$



A **theorem** is a formal rule or formula. **Pythagoras' theorem** was discovered by the ancient Greek mathematician Pythagoras. This rule is used to find the length of one side of a right-angled triangle when the other 2 sides are known.

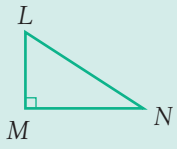
Alamy Stock Photo/Ilan Rosen

This artwork of Pythagoras and his theorem is from the University of Coimbra in Portugal.

Foundation Standard Complex

Example 4

Write Pythagoras' theorem for $\triangle LNM$.



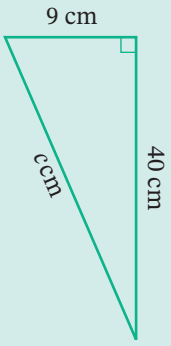
Solution

The hypotenuse is LN, the 2 shorter sides are LM and MN.

So Pythagoras' theorem is $LN^2 = LM^2 + MN^2$

Example 5

Find the value of c in this triangle.



Solution

c cm is the length of the hypotenuse. Using Pythagoras' theorem:

$c^2 = a^2 + b^2$

$c^2 = 9^2 + 40^2$ Substitute the lengths of the 2 shorter sides.

$= 1681$

$c = \sqrt{1681}$ Use the square root to find c.

$= 41$

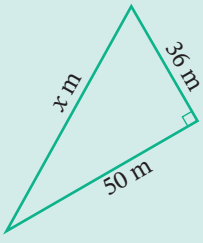
Check: A hypotenuse of 41 cm looks reasonable from the diagram: it is the longest side and a little longer than 40 cm.

Example 6

Find the value of x in this triangle:

a as a surd

b correct to 2 decimal places.



Solution

a Write Pythagoras' theorem.

$c^2 = a^2 + b^2$

$x^2 = 50^2 + 36^2$ The hypotenuse in this question is x.

$= 3796$ Substitute the lengths of the 2 shorter sides.

$x = \sqrt{3796}$

Because the question asks for a surd, leave the answer in square root ($\sqrt{}$) form.

b $x = \sqrt{3796}$ From part **a**.

$= 61.6116\ldots$

≈ 61.61 *Check*: From the diagram, this answer looks reasonable.

Note: The surd answer is more exact than the decimal answer because it is not rounded.

1.03

Pythagoras' theorem 1

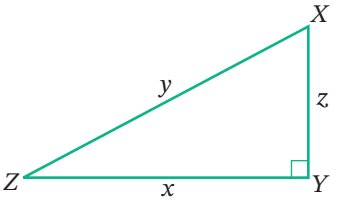
Pythagoras' theorem 2

EXERCISE 1.03 ANSWERS ON P. 544

Finding the hypotenuse U F C

EXAMPLE 4

1 What is Pythagoras' theorem for this triangle?
Select the correct answer **A**, **B**, **C** or **D**. C



A $z^2 = x^2 + y^2$

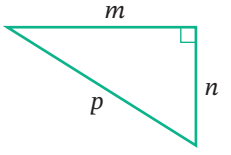
B $ZY^2 = ZX^2 + XY^2$

C $XY^2 = ZY^2 + ZX^2$

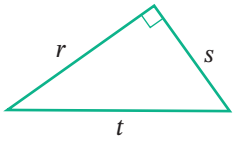
D $y^2 = x^2 + z^2$

2 Write Pythagoras' theorem for each right-angled triangle. C

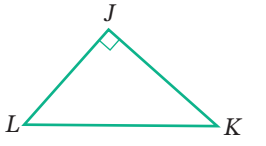
a



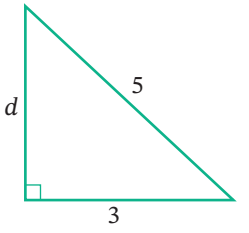
b



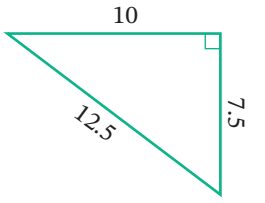
c



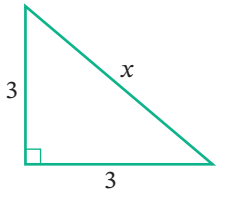
d



e



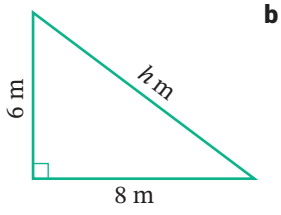
f



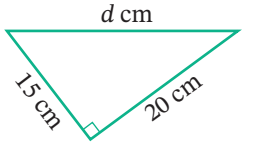
EXAMPLE 5

3 Find the value of each variable.

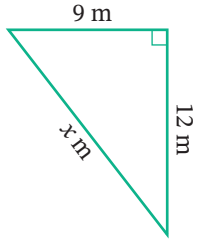
a



b



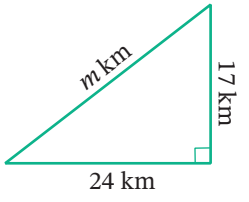
c



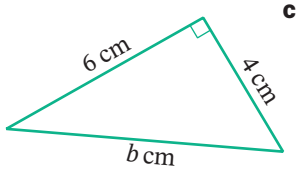
EXAMPLE 6

4 Find the value of each variable as a surd.

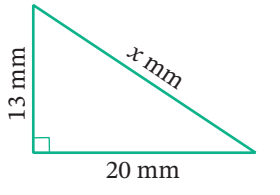
a



b



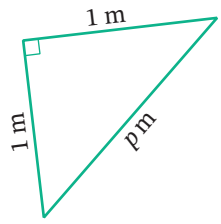
c



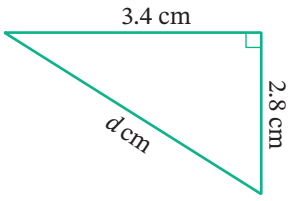
Foundation Standard Complex

9780170453141

d

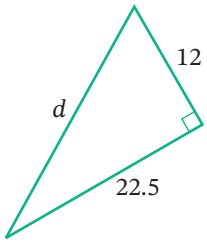


e



5 For each triangle in question **4**, find correct to one decimal place the value of each variable.

6 In this triangle, what is the value of d? Select **A**, **B**, **C** or **D**.

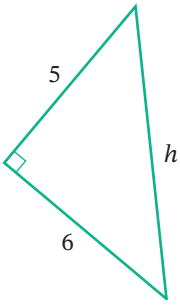


A 22.8 **B** 12.9

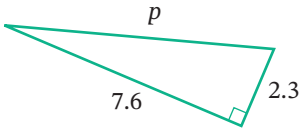
C 34.5 **D** 25.5

7 Find the value of each variable, correct to 2 decimal places.

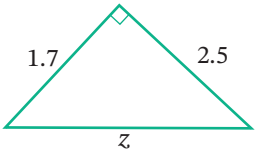
a



b



c



8 Find the length of the hypotenuse of a right-angled triangle with shorter sides of length 7.5 cm and 10 cm.

9 Copy and complete each equation. **C**

a $24^2 + 32^2 = \_\_\_\_^2$ **b** $36^2 + 15^2 = \_\_\_\_^2$

c $9^2 + 40^2 = \_\_\_\_^2$ **d** $21^2 + \_\_\_\_^2 = 35^2$

e $60^2 + \_\_\_\_^2 = 61^2$ **f** $\_\_\_\_^2 + 15^2 = 17^2$

g $\_\_\_\_^2 + 24^2 = 51^2$ **h** $\_\_\_\_^2 + \_\_\_\_^2 = 10^2$

i $\_\_\_\_^2 + \_\_\_\_^2 = 15^2$

Did you know?

Pythagoras

Pythagoras was a Greek philosopher and mathematician. He was born on the island of Samos, part of ancient Greece, and lived from 570 BCE to 490 BCE. He was the first person to set down a mathematical proof for his famous theorem, even though it was known and used long before his time.

The Pythagoreans were a group of men who followed Pythagoras' philosophical ideas and worked on mathematical problems. They formed a secret society known as the Pythagoreans. Apparently, they were so upset about the discovery of surds that they tried to keep it a secret. Hippasus, one of the Pythagoreans, was drowned for revealing the secret to outsiders.

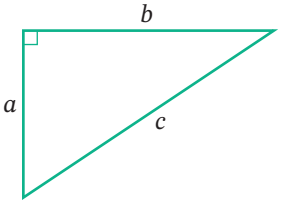
Which American president published a proof of Pythagoras' theorem? When did he do this?

Shutterstock.com/OmoPhoto

Statue of Pythagoras in modern Samos, Greece.

Technology

Finding the hypotenuse



In this activity, we will create a spreadsheet to calculate the length of the hypotenuse, c, of a right-angled triangle, given the lengths of the other two sides (a and b).

1 Enter the headings and values shown.

	A	B	C	D	E
1	a	b	a^2+b^2	c	
2	3	4			
3	5	12			
4	6	8			
5	7	24			
6					
7					

Remember: a^2 + b^2 means $a^2 + b^2$.

2 Enter into cell C2 the formula **=A2^2+B2^2**, and use **Fill Down** to copy this formula into cells C3 to C5.

3 To calculate the length of each hypotenuse in column D, we need to find the square root of the values in column C, so in cell D2, enter **=sqrt(C2)**. Then **Fill Down** to copy this formula into cells D3 to D5.

4 Use your spreadsheet to check your answers to questions **3** to **6** of Exercise 1.03.

Finding a shorter side

1.04

Pythagoras' theorem can also be used to find the length of one of the shorter sides in a right-angled triangle if the hypotenuse and the other side are given. In this case, we *subtract* the square of the length of the known shorter side from the square of the length of the hypotenuse.

Alamy Stock Photo/Myrleen Pearson

Pythagoras' theorem

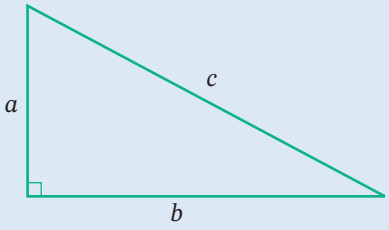
Pythagoras' theorem

Finding a shorter side

To find the length of a shorter side, a, in a right-angled triangle with hypotenuse c and other shorter side b, use Pythagoras' theorem in the form:

$$a^2 = c^2 - b^2$$

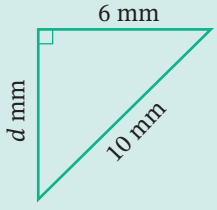
Remember the correct order for $c^2 - b^2$, the hypotenuse is squared first, otherwise we will get a negative number.



Example 7

Pythagoras' theorem 2

Find the value of d in this triangle.



Solution

d is one of the shorter sides.

$a^2 = c^2 - b^2$

$d^2 = 10^2 - 6^2$ (Hypotenuse)^2 − (other side)^2

$= 64$

$d = \sqrt{64}$ Take the square root to find d.

$= 8$ *Check*: From the diagram, the answer $d = 8$ looks reasonable.

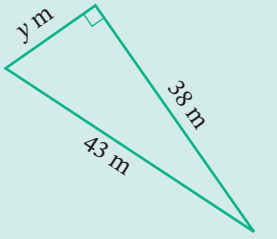
Pythagoras' theorem 1

Example 8

Find the value of y in this triangle:

a as a surd

b correct to 2 decimal places.



Solution

a $a^2 = c^2 - b^2$

$y^2 = 43^2 - 38^2$ (Hypotenuse)^2 – (other side)^2

$= 405$

$y = \sqrt{405}$ Surd answer

b $y = \sqrt{405}$ from part **a**

$= 20.1246\ldots$

≈ 20.12

Check: From the diagram, this answer appears to be reasonable.

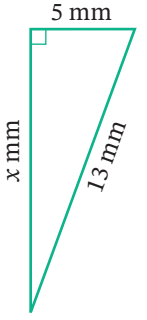
EXERCISE 1.04 ANSWERS ON P. 544

Finding a shorter side U F R

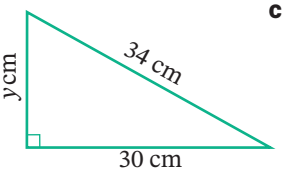
EXAMPLE 7

1 Find the value of each variable.

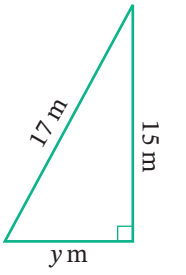
a



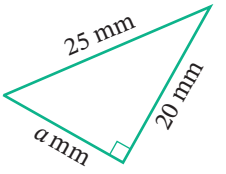
b



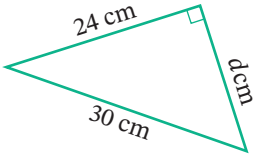
c



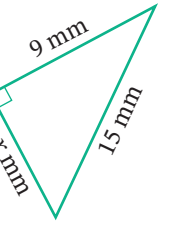
d



e



f



Foundation | Standard | Complex

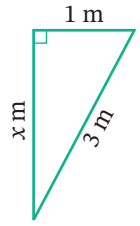
9780170453141

EXAMPLE 8

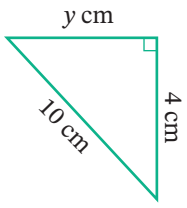
1.04

2 Find the value of each variable, giving your answer in surd form.

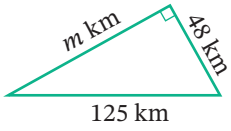
a



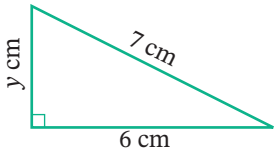
b



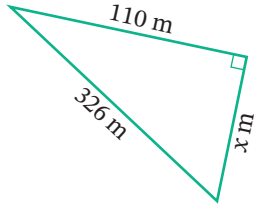
c



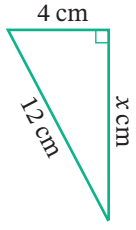
d



e

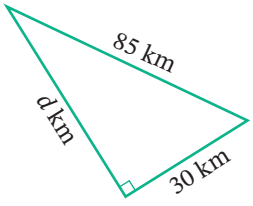


f



3 Find the value of each variable in question **2**, correct to one decimal place.

4 What is the value of d? Select the closest answer **A**, **B**, **C** or **D**.



A 74.16

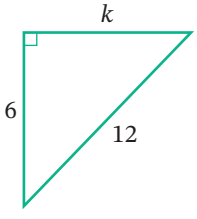
B 79.53

C 90.14

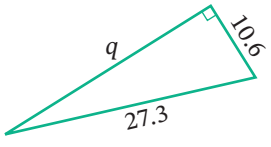
D 127.50

5 Find the value of each variable, correct to 2 decimal places.

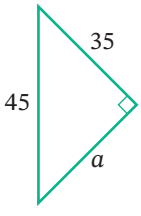
a



b



c



6 Find the length of the unknown side of a right-angled triangle with a hypotenuse of length 32.5 cm and one of the short sides of length 12.5 cm.

7 What could be the lengths of the other 2 sides of a right-angled triangle if: R

a one of the shorter sides is $\sqrt{7}$ cm long?

b one of the shorter sides is 7 cm long?

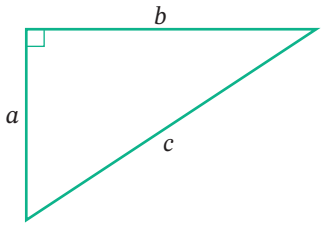
c the hypotenuse is $\sqrt{20}$ cm long?

d the hypotenuse is 20 cm long?

Foundation | Standard | Complex

Technology

Finding a shorter side



In this activity, we will create a spreadsheet to calculate the length of a shorter side, a, of a right-angled triangle, given the lengths of the hypotenuse (c) and the other side (b).

1 Enter the headings and values shown below.

	A	B	C	D
1	c	b	c^2 - b^2	a
2	25	20		
3	37	12		
4	11	7		
5	29	21		

2 Enter a formula into cell C2 to calculate $c^2 - b^2$, which is 225, and use **Fill Down** to copy this formula into cells C3 to C5.

3 To calculate the length of the shorter side (a) in column D, we need to find the square root of $c^2 - b^2$. Enter a formula into cell D2 to calculate the square root of cell C2, which is 15, and use **Fill Down** to copy this formula into cells D3 to D5.

4 Use your spreadsheet to check your answers to Exercise 1.04.

Mental skills 1: Maths without calculators ANSWERS ON P. 544

Squaring a number ending in 5 or 1

The square of a number ending in 5 always ends in 25.
For example, $35^2 = 1225$ and $105^2 = 11\,025$.
A mental calculation trick requires 3 easy steps:

- delete the 5 from the number
- multiply the remaining number by the next consecutive number
- write '25' at the end of the product.

1 Study each example.

a To calculate 35^2:

- deleting the 5 from 35 leaves 3
- multiply 3 by the next consecutive number: $3 \times 4 = 12$
- write '25' at the end: 1225

$35^2 = 1225$

b To calculate 105^2:

- deleting the 5 from 105 leaves 10
- multiply 10 by the next consecutive number: $10 \times 11 = 110$
- write 25 at the end: 11 025

$105^2 = 11\,025$

2 Now calculate each square number.

a	25^2	**b**	55^2	**c**	45^2	**d**	85^2	**e**	115^2	**f**	7.5^2
g	95^2	**h**	195^2	**i**	1.5^2	**j**	65^2	**k**	155^2	**l**	245^2

The square of a number ending in 1 always ends in 1.
For example, $41^2 = 1681$ and $71^2 = 5041$.
A mental calculation trick requires 3 steps:

- round the number down to the nearest 10 (by subtracting 1) to make a new number
- square the new number
- to your answer, add the new number and its next consecutive number

3 Study each example.

a To calculate 41^2:

- round 41 down to 40
- square 40: $40^2 = 1600$
- add 40 and 41: $1600 + 40 + 41 = 1681$

$41^2 = 1681$

b To calculate 71^2:

- round 71 down to 70
- square 70: $70^2 = 4900$
- $4900 + 70 + 71 = 5041$

$71^2 = 5041$

4 Now calculate each square number.

a	21^2	**b**	101^2	**c**	31^2	**d**	91^2	**e**	5.1^2
f	81^2	**g**	61^2	**h**	201^2	**i**	1.1^2	**j**	4.1^2

Hypotenuse or shorter side? 1.05

Let's solve some mixed problems that require using Pythagoras' theorem. To find the length of any unknown side in a right-angled triangle, follow these steps.

1 Decide whether the unknown side is the **hypotenuse** or one of the **shorter sides**.

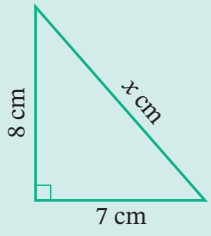
2 To find the hypotenuse, use $c^2 = a^2 + b^2$ and *add*.
To find a shorter side, use $a^2 = c^2 - b^2$ and *subtract*.

3 If a **surd answer** is required, leave the answer in square root ($\sqrt{\ }$) form. Otherwise, calculate the value correct to the required number of decimal places.

Pythagoras' theorem
Finding an unknown side
Pythagoras 1
Pythagoras 2

Example 9

Find the value of x as a surd.



Solution

x is the hypotenuse.

$c^2 = a^2 + b^2$

$x^2 = 7^2 + 8^2$
$= 113$

$x = \sqrt{113}$

As a surd.

Example 10

Find the value of n, correct to one decimal place.

Solution

n is a shorter side.

$a^2 = c^2 - b^2$

$n^2 = 4.3^2 - 1.9^2$ (Hypotenuse)^2 − (other side)^2

$= 14.88$

$n = \sqrt{14.88}$ Find the square root.

$= 3.85746\ldots$

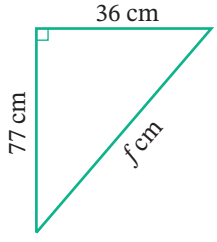
≈ 3.9

EXERCISE 1.05 ANSWERS ON P. 544

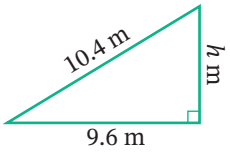
Hypotenuse or shorter side? U F R C

1 For each triangle, state whether the unknown side is the hypotenuse (H) or one of the shorter sides (S). R C

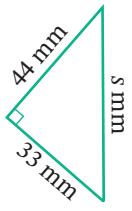
a



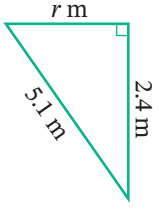
b



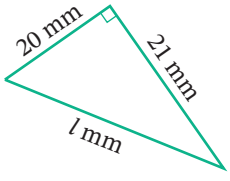
c



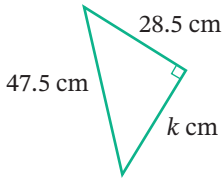
d



e



f

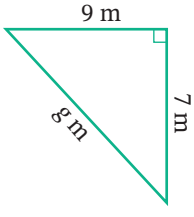


2 Find the value of each variable in each triangle in question **1**.

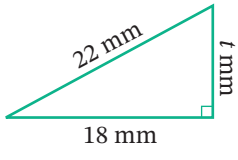
EXAMPLE 9

3 Find the value of each variable as a surd.

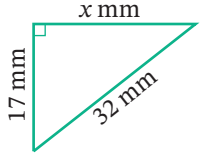
a



b



c



Foundation Standard Complex

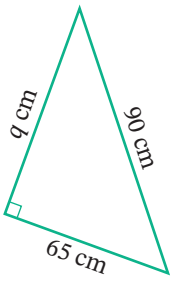
9780170453141

EXAMPLE 10

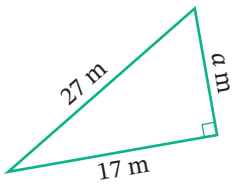
1.05

4 Find the value of each variable correct to one decimal place.

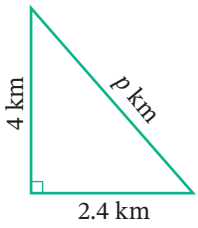
a



b



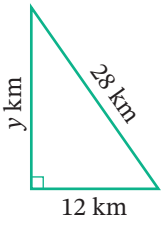
c



5 What is the value of y? Select the closest answer **A**, **B**, **C** or **D**.

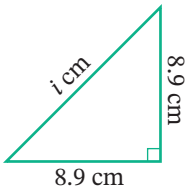
A 25.29 **B** 25.30

C 30.47 **D** 30.46

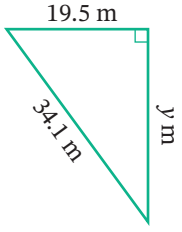


6 Find the value of each variable correct to 2 decimal places.

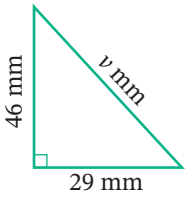
a



b

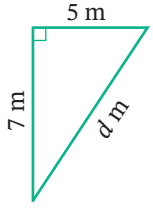


c

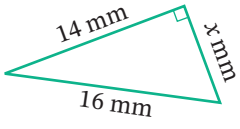


7 Find the value of each variable as a surd.

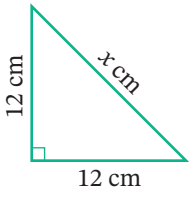
a



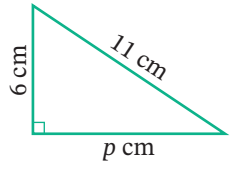
b



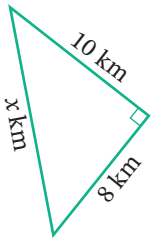
c



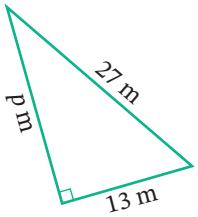
d



e

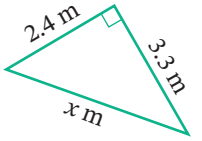


f

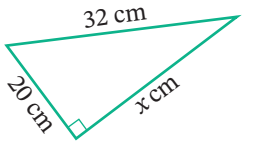


Foundation Standard Complex

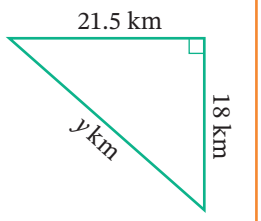
g



h



i



1.06 Testing for right-angled triangles

WS Testing for right-angled triangles

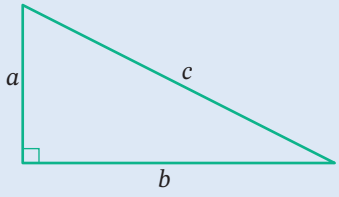
Pythagoras' theorem tester

Pythagoras' theorem can be used to test whether a triangle is right-angled.

Testing for right-angled triangles

If the sides of a triangle have lengths a, b and c, where c is the largest, and they follow the formula $c^2 = a^2 + b^2$, then the triangle must be right-angled.

The right angle is opposite the longest side, c.



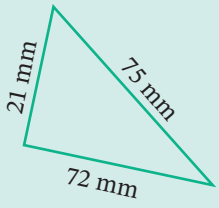
This is called the **converse** of Pythagoras' theorem. The word 'converse' means to 'turn around' or 'reverse'. So the converse of a theorem is the theorem written 'back-to-front'.

Testing for right-angled triangles

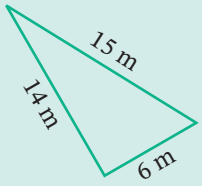
Example 11

Test whether each triangle is right-angled.

a



b



Solution

a $75^2 = 5625$ — Squaring the longest side.

$21^2 + 72^2 = 5625$ — Squaring the 2 shorter sides, and adding.

$\therefore 75^2 = 21^2 + 72^2$ — The 3 sides follow $c^2 = a^2 + b^2$.

$\therefore$ The triangle is right-angled (with the right angle opposite the 75 mm side).

$\therefore$ is the symbol for 'therefore'

Foundation Standard Complex

9780170453141

b $15^2 = 225$ Squaring the longest side.

$6^2 + 14^2 = 232$ Squaring the 2 shorter sides, and adding.

$\therefore 15^2 \neq 6^2 + 14^2$ The 3 sides do not follow $c^2 = a^2 + b^2$.

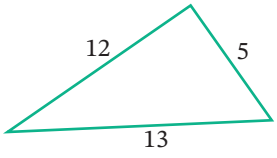
$\therefore$ The triangle is not right-angled.

EXERCISE 1.06 ANSWERS ON P. 544

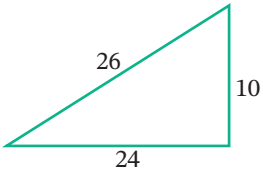
Testing for right-angled triangles U F PS R C

1 Copy each triangle, test whether it is right-angled, then mark the right angle on the triangle. R

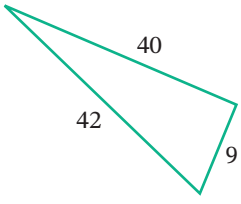
a



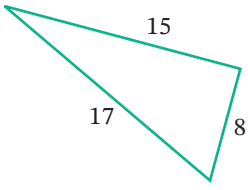
b



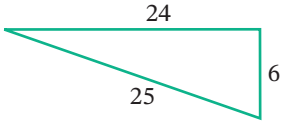
c



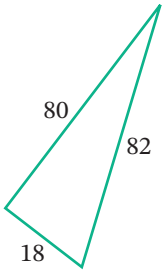
d



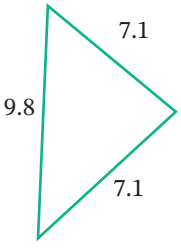
e



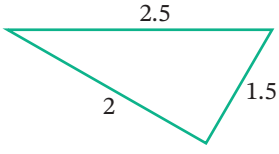
f



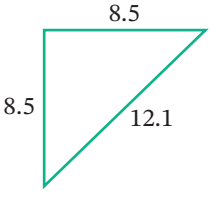
g



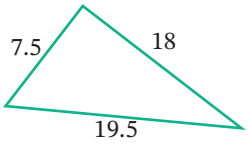
h



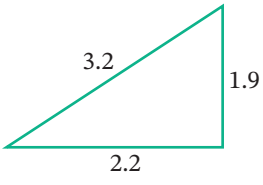
i



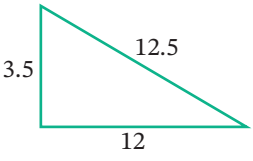
j



k



l



2 Jane and Mark are building the framework for a rectangular concrete slab. They want to check if it has right angles at the corners. They measure the sides and the diagonal. The sides are 3.6 m and 4.8 m. The diagonal is 6 m. PS R C

a Draw a diagram showing this information.

b Have they constructed their framework with right angles at the corners? Justify your answer with a calculation.

1.07 Pythagorean triads


Pythagorean triples

A **Pythagorean triad** or **Pythagorean triple** is any group of 3 numbers that follow Pythagoras' theorem, for example, (3, 4, 5) or (2.5, 6, 6.5). The word **triad** means a group of 3 related items ('tri-' means 3).


Pythagorean triples

Pythagorean triads

(a, b, c) is a **Pythagorean triad** if

$$c^2 = a^2 + b^2.$$

Any multiple of (a, b, c) is also a Pythagorean triad.

Example 12

Test whether (5, 12, 13) is a Pythagorean triad.

Solution

$13^2 = 169$ Squaring the largest number.

$5^2 + 12^2 = 169$ Squaring the 2 smaller numbers and adding them.

$\therefore 13^2 = 5^2 + 12^2$ These 3 numbers follow Pythagoras' theorem.

$\therefore$ (5, 12, 13) is a Pythagorean triad.

Example 13

{3, 4, 5} is a Pythagorean triad. Create other Pythagorean triads by multiplying (3, 4, 5) by:

a 2 **b** 9 **c** $\frac{1}{2}$

Solution

a $2 \times (3, 4, 5) = (6, 8, 10)$

Checking: $10^2 = 100$

$6^2 + 8^2 = 100$

$\therefore 10^2 = 6^2 + 8^2$

$\therefore$ (6, 8, 10) is a Pythagorean triad.

b $9 \times (3, 4, 5) = (27, 36, 45)$

Checking: $45^2 = 2025$

$27^2 + 36^2 = 2025$

$\therefore 45^2 = 27^2 + 36^2$

$\therefore$ (27, 36, 45) is a Pythagorean triad.

c $\frac{1}{2} \times (3, 4, 5) = (1.5, 2, 2.5)$

Checking: $2.5^2 = 6.25$

$1.5^2 + 2^2 = 6.25$

$\therefore 2.5^2 = 1.5^2 + 2^2$

$\therefore$ (1.5, 2, 2.5) is a Pythagorean triad.

EXERCISE 1.07 ANSWERS ON P. 545

Pythagorean triads U F R

1 Test whether each triad is a Pythagorean triad. R

EXAMPLE 12

a (8, 15, 17) **b** (10, 24, 26) **c** (30, 40, 50)
d (5, 7, 9) **e** (9, 40, 41) **f** (4, 5, 9)
g (11, 60, 61) **h** (7, 24, 25) **i** (15, 114, 115)

2 Which of the following is a Pythagorean triad? Select the correct answer **A**, **B**, **C** or **D**. R

A (4, 6, 8) **B** (5, 10, 12) **C** (6, 7, 10) **D** (20, 48, 52)

3 If you used the technology worksheet **Pythagoras' theorem tester** on page 20 to create a spreadsheet, then use it to check your answers to questions **1** and **2**.

4 For each Pythagorean triad, create another Pythagorean triad by multiplying each number in the triad by:

EXAMPLE 13

i a whole number **ii** a fraction **iii** a decimal.

a (5, 12, 13) **b** (8, 15, 17) **c** (30, 40, 50) **d** (7, 24, 25)

Check that each answer follows Pythagoras' theorem. R

5 Pythagoras developed a formula for finding Pythagorean triads (a, b, c).
If one number in the triad is a, the formulas for the other 2 numbers are $b = \frac{1}{2}(a^2 - 1)$ and $c = \frac{1}{2}(a^2 + 1)$. R

a If $a = 5$, use the formulas to find the values of b and c.

b Hence show that (a, b, c) is a Pythagorean triad.

6 Use the formulas to find Pythagorean triads for each value of a.

a $a = 7$ **b** $a = 11$ **c** $a = 15$ **d** $a = 4$
e $a = 9$ **f** $a = 19$ **g** $a = 10$ **h** $a = 51$

7 There are many other formulas for creating Pythagorean triads. Use the Internet to search for some of them and test that they are correct.

Shutterstock.com/Hatsaniuk

1.08 Pythagoras' theorem problems

Pythagoras' theorem has many practical applications, from finding the diagonal length of a TV screen to calculating the distance a ship has sailed.

WS Pythagoras' theorem in 2D and 3D

WS TV screens

Example 14

The size of a TV screen is described by the length of its diagonal. If a flat-screen TV is 89 cm wide and 50 cm high, what is the size of its screen? Answer to the nearest centimetre.

Shutterstock.com/Ruslan Ivantsov

Solution

Let the diagonal length be d cm.

$d^2 = 89^2 + 50^2$ — d is the hypotenuse.

$= 10\,421$

$d = \sqrt{10\,421}$

$= 102.0832\ldots$

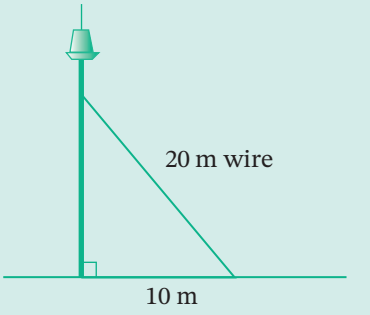
≈ 102

The size of the screen is 102 cm.

Pythagoras' theorem problems

Example 15

A tower is supported by a wire that is 20 m long and attached to the ground 10 m from the base of the tower. How high does the wire reach up the tower? Answer correct to the nearest 0.1 m.



Solution

Let h m be how high the wire reaches up the tower.

$h^2 = 20^2 - 10^2$ — h is a shorter side.

$= 300$

$h = \sqrt{300}$

$= 17.3205$

≈ 17.3

The wire reaches 17.3 m up the tower.

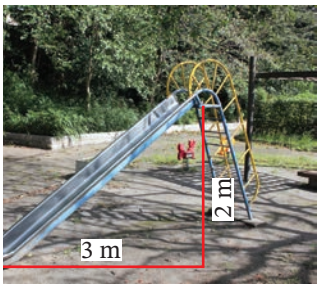
EXERCISE 1.08 ANSWERS ON P. 545

Pythagoras' theorem problems U F PS R

EXAMPLE 14

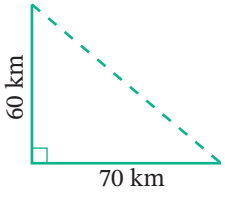
1.08

1 Find the length of this playground slide, correct to 2 decimal places.



iStock.com/Masahito Ueno

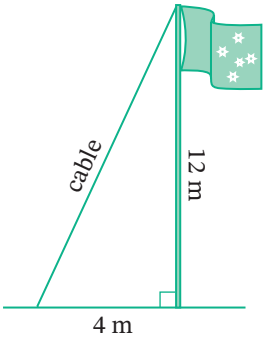
2 A ship sails 60 km south and then 70 km east. How far is it from its starting point, correct to one decimal place?



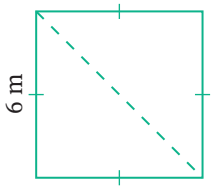
3 What is the length of a cable used to stabilise a flagpole that is 12 metres high, if the cable is secured to the ground 4 m from the base of the flagpole? Select the closest answer **A**, **B**, **C** or **D**.

A 11.3 m **B** 12.6 m

C 16 m **D** 80 m



4 Find the length of the diagonal of a square with sides of length 6 metres. Give your answer as a surd.



5 A firefighter places a ladder on a window sill 9.5 m above the ground. If the foot of the ladder is 1.8 m from the wall, how long is the ladder, correct to one decimal place?

Alamy Stock Photo/Willows Photos UK

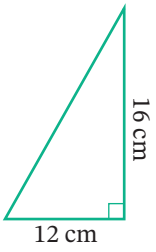
Foundation Standard Complex

EXAMPLE 15

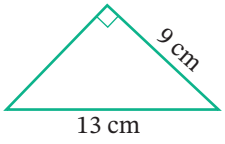
6 Find the height of a TV screen with a 140 cm diagonal if its length is 122 cm. Give your answer correct to one decimal place. PS

7 By first using Pythagoras' theorem to find the length of the unknown side, find the perimeter of each shape below, correct to 2 decimal places where necessary.

a

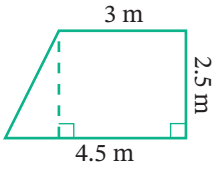


b

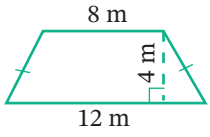


c

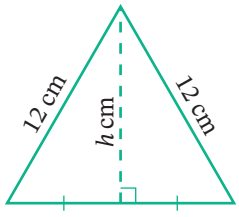
d



e

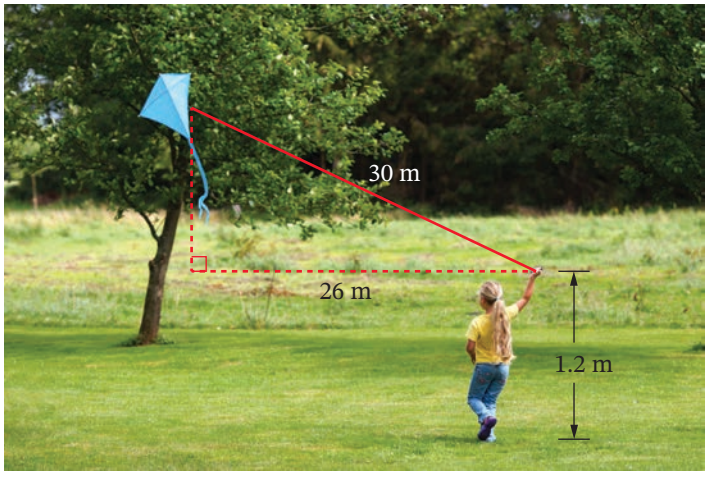


8 An equilateral triangle has sides of length 12 cm. Find its perpendicular height, h, correct to 2 decimal places. PS R



9 A rope is tied to the top of the 6 metre wall of a tent and tied to a peg in the ground. The peg is 2 m from the bottom of the tent. How long is the rope, correct to 2 decimal places? PS R

10 Olga holds a kite string 1.2 m above the ground. How high is the kite above the ground, to the nearest metre? PS R



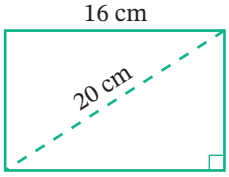
Shutterstock.com/Ramona Heim

Foundation Standard Complex

9780170453141

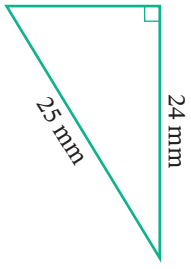
11 Find the area of each shape below, correct to one decimal place where necessary. R

a

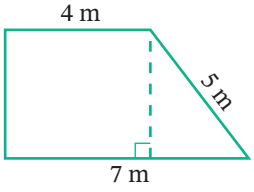


b

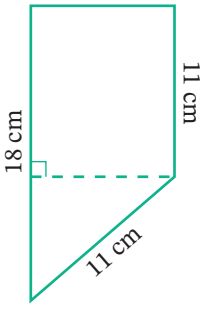
c



d



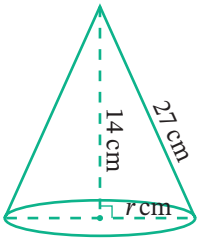
e



12 This baseball diamond is a square shape of length 27 m. What is the distance from the home plate to second base, correct to the nearest metre?

Shutterstock.com/Creative Droneworks

13 The slant height of a cone is 27 cm and its vertical height is 14 cm. Find the radius of the base circle, correct to the nearest centimetre.



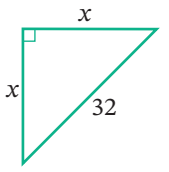
14 A ladder 5 m long leans against a wall, with its base 2 m from the bottom of the wall. How far does the ladder reach up the wall, correct to nearest centimetre? PS R

Foundation | Standard | Complex

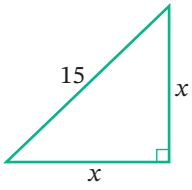
Power plus ANSWERS ON P. 545

1 Find the value of x in each triangle, correct to one decimal place.

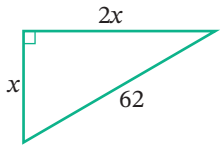
a



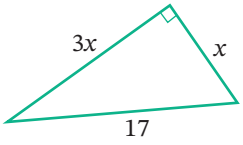
b



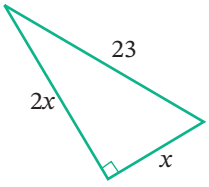
c



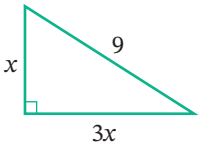
d



e



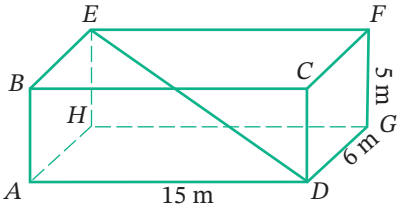
f



2 Find, correct to one decimal place, the length of:

a HD

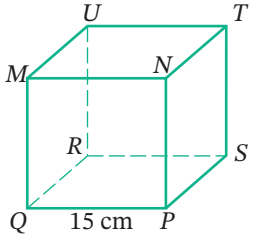
b DE



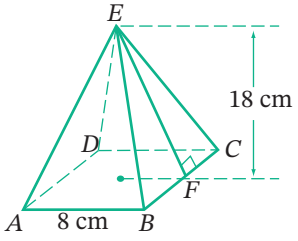
3 For this cube of side length 15 cm, find, correct to one decimal place, the length of:

a QS, the diagonal of the base

b QT, the diagonal of the cube



4 For this square pyramid, find the slant height EF correct to one decimal place.



9780170453141

CHAPTER 1 REVIEW

Language of maths

area	converse	diagonal	formula
hypotenuse	irrational	perimeter	Pythagoras
right-angled	shorter	side	square root
surd	theorem	triad	unknown

1 Who was **Pythagoras** and which country did he come from?

2 Describe the **hypotenuse** of a right-angled triangle in 2 ways.

3 What is another word for 'theorem'?

4 For what type of triangle is **Pythagoras' theorem** used?

5 What is a **surd**?

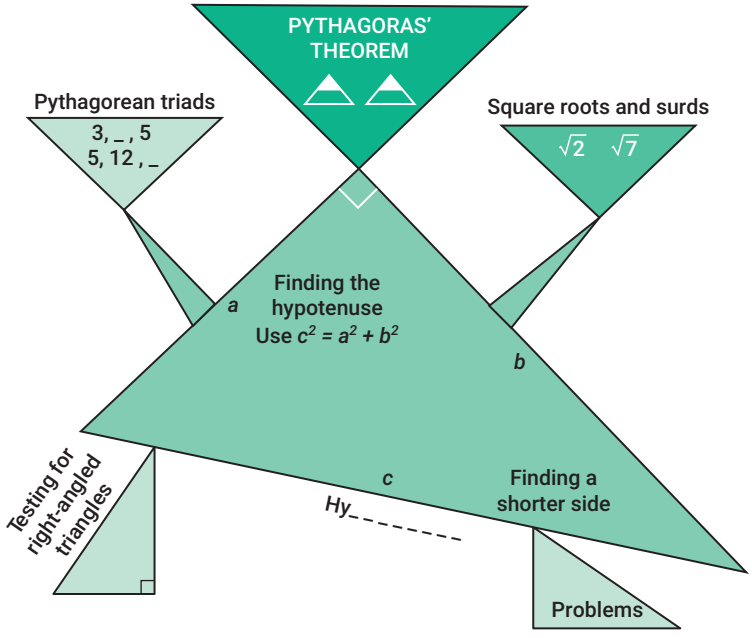
6 What is the name given to a set of 3 numbers that follows Pythagoras' theorem?

Topic summary

- How relevant do you think Pythagoras' theorem is to our world? Give reasons for your answer.
- Give 3 examples of careers that would use Pythagoras' theorem.
- What did you find especially interesting about this topic?
- Is there any section of this topic that you found difficult? Discuss any problems with your teacher or a friend.

WS
Mind map: Pythagoras' theorem

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



TEST YOURSELF 1 ANSWERS ON P. 545

1.01

1 Evaluate each expression, correct to 2 decimal places.

a $\sqrt{203}$ **b** $\sqrt{15^2+6^2}$ **c** $\sqrt{7^2-4^2}$ **d** $\sqrt{3.4^2+5.2^2}$

1.01

2 Select all the surds from this list of square roots.

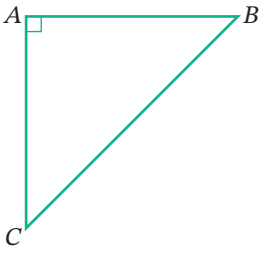
$\sqrt{25}$ $\sqrt{104}$ $\sqrt{96}$ $\sqrt{256}$ $\sqrt{12}$

$\sqrt{169}$ $\sqrt{121}$ $\sqrt{45}$ $\sqrt{729}$ $\sqrt{88}$

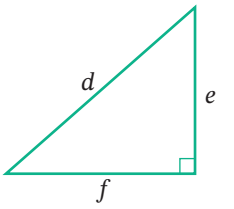
1.02

3 For each right-angled triangle, name the hypotenuse and write Pythagoras' theorem.

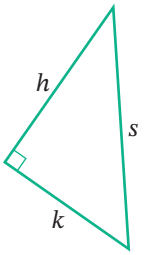
a



b



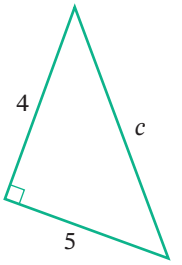
c



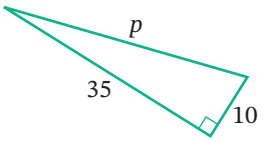
1.03

4 Find the value of each variable, giving your answer as a surd.

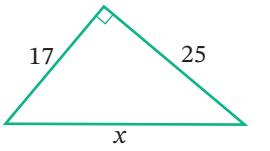
a



b



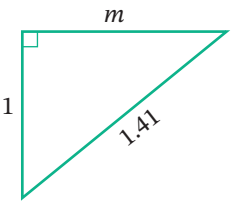
c



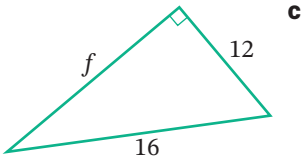
1.04

5 Find the value of each variable, giving your answer correct to 2 decimal places.

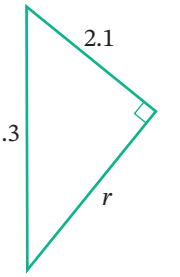
a



b



c

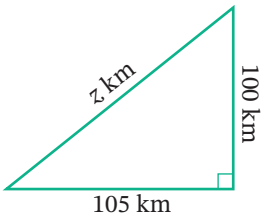


Foundation Standard Complex

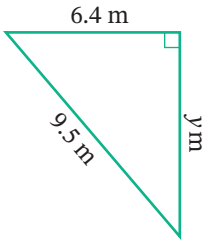
9780170453141

6 Find the value of each variable, correct to one decimal place. 1.05

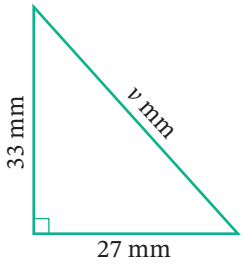
a



b

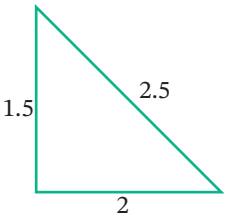


c

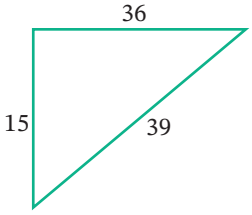


7 Test whether each triangle is right-angled. If the triangle is right-angled, sketch it showing the right angle. 1.06

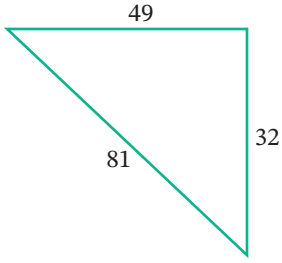
a



b



c



8 Test whether each triad is a Pythagorean triad. 1.07

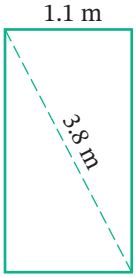
a (15, 20, 25) **b** (11, 14, 20) **c** (20, 21, 29) **d** (2.5, 6, 6.5)

9 Find the length of the longest pencil that can fit inside this pencil case. Answer to the nearest mm. 1.08

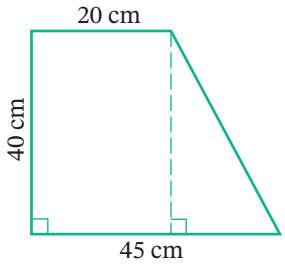
Shutterstock.com/Creative Images

10 Find the perimeter of each shape, correct to one decimal place. 1.08

a



b



Foundation Standard Complex